

Ba

## 式の計算編

## 正答・解説

## 正 答

問題番号	解答記号	正解
1.	$\frac{1}{(1+\sqrt{6})-\alpha}$	$\frac{1}{(1+\sqrt{6})-3}$
	$\frac{\sqrt{6}-\beta}{\gamma}$	$\frac{\sqrt{6}-2}{4}$
	$\epsilon + \delta \sqrt{6}$	$2+2\sqrt{6}$
	$\kappa - \sqrt{6}$	$\frac{4-\sqrt{6}}{2}$

問題番号	解答記号	正解
2.	$\alpha$	2
	$\beta (\epsilon + \delta \sqrt{6})$	$2(-1+\sqrt{6})$
	$\kappa (\kappa - \sqrt{6})$	$8(3-\sqrt{6})$
	ケコ	16
	$a^4 + \alpha a^3 - \beta a^2 + \gamma a + \delta = 0$	$a^4 + 4a^3 - 16a^2 + 8a + 4 = 0$

## 解 説

1.

$$\begin{aligned}
 AB &= \frac{1}{1+\sqrt{6}+\sqrt{3}} \cdot \frac{1}{1+\sqrt{6}-\sqrt{3}} \\
 &= \frac{1}{(1+\sqrt{6})^2 - (\sqrt{3})^2} = \frac{1}{(1+\sqrt{6})^2 - 3} \\
 &= \frac{1}{1+2\sqrt{6}+6-3} = \frac{1}{4+2\sqrt{6}} \\
 &= \frac{1}{2(2+\sqrt{6})} = \frac{2-\sqrt{6}}{2(4-6)} = \frac{2-\sqrt{6}}{2 \cdot (-2)} \\
 &= \frac{2-\sqrt{6}}{-4} = \underline{\frac{\sqrt{6}-2}{4}}
 \end{aligned}$$

$$\frac{1}{A} = 1 + \sqrt{3} + \sqrt{6}, \quad \frac{1}{B} = 1 - \sqrt{3} + \sqrt{6} \text{ だから}$$

$$\begin{aligned}
 \frac{1}{A} + \frac{1}{B} &= 1 + \sqrt{3} + \sqrt{6} + 1 - \sqrt{3} + \sqrt{6} \\
 &= \underline{2 + 2\sqrt{6}}
 \end{aligned}$$

$$\frac{1}{A} + \frac{1}{B} = \frac{B}{AB} + \frac{A}{AB} = \frac{A+B}{AB} \text{ より、}$$

$$\begin{aligned}
 A+B &= \left( \frac{1}{A} + \frac{1}{B} \right) \cdot AB \\
 &= (2 + 2\sqrt{6}) \cdot \frac{\sqrt{6}-2}{4} \\
 &= \frac{(\sqrt{6}+1)(\sqrt{6}-2)}{4} \\
 &= \frac{6-\sqrt{6}-2}{2} = \underline{\frac{4-\sqrt{6}}{2}}
 \end{aligned}$$

2.

$$(1) \quad a = \frac{1+\sqrt{3}}{1+\sqrt{2}}, \quad b = \frac{1-\sqrt{3}}{1-\sqrt{2}} \text{ より}$$

$$ab = \frac{1+\sqrt{3}}{1+\sqrt{2}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{2}} = \frac{1-3}{1-2} = \frac{-2}{-1} = \underline{2}$$

$$\begin{aligned}
 a+b &= \frac{1+\sqrt{3}}{1+\sqrt{2}} + \frac{1-\sqrt{3}}{1-\sqrt{2}} \\
 &= \frac{(1+\sqrt{3})(1-\sqrt{2})}{1-2} + \frac{(1-\sqrt{3})(1+\sqrt{2})}{1-2} \\
 &= -(1-\sqrt{2}+\sqrt{3}-\sqrt{6}) - (1+\sqrt{2}-\sqrt{3}-\sqrt{6}) \\
 &= -1 + \sqrt{2} - \sqrt{3} + \sqrt{6} - 1 - \sqrt{2} + \sqrt{3} + \sqrt{6} \\
 &= -2 + 2\sqrt{6} = \underline{2(-1+\sqrt{6})}
 \end{aligned}$$

$$a^2 + b^2 = (a+b)^2 - 2ab$$

$$= (-2 + 2\sqrt{6})^2 - 2 \cdot 2$$

$$= 4 - 8\sqrt{6} + 24 - 4$$

$$= 24 - 8\sqrt{6} = \underline{8(3-\sqrt{6})}$$

$$(2) \quad a^2 + b^2 + 4(a+b) = (24 - 8\sqrt{6}) + 4(-2 + 2\sqrt{6})$$

$$= 24 - 8\sqrt{6} - 8 + 8\sqrt{6} = \underline{16}$$

$$ab = 2 \text{ より、 } b = \frac{2}{a} \text{ を}$$

$$a^2 + b^2 + 4(a+b) = 16 \sim \text{代入}$$

$$a^2 + \frac{4}{a^2} + 4a + \frac{8}{a} = 16$$

$$\text{両辺} \times a^2$$

$$a^4 + 4 + 4a^3 + 8a = 16a^2$$

$$\underline{a^4 + 4a^3 - 16a^2 + 8a + 4 = 0}$$