第3回

[1]

[1]

(1) $\log_2 x = \log_4(2x+3)$ —①

$x>0$, $2x+3>0$

$x > -\frac{3}{2}$

よって $x > \underline{0}$ ア

$\log_4(2x+3) = \frac{\log_2(2x+3)}{\log_2 4}$

$= \underline{\frac{1}{2}}\log_2(2x+3)$ イウ

①より

$\log_2 x = \frac{1}{2}\log_2(2x+3)$

$= \log_2(2x+3)^{\frac{1}{2}}$

$x = (2x+3)^{\frac{1}{2}}$

$x^{\underline{2}} = 2x+3$ エ

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x = 3, -1$

$x>0$ より $x = \underline{3}$ オ

(2) $k>0$

$\log_2 x = \log_4(2x+3)+k$ —②

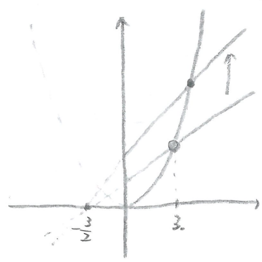
$= \log_2(2x+3)^{\frac{1}{2}} + \log_2 2^k$

$= \log_2\{(2x+3)^{\frac{1}{2}}\cdot 2^k\}$

$x = (2x+3)^{\frac{1}{2}}\cdot 2^k$

$x^2 = (2x+3)\cdot 2^{2k}$

$x^2 = \underline{4^k}(2x+3)$ カ



$x^2=2x+3$ の解は $x=3$

よって ②の解の範囲は $x > \underline{3}$ キ

$x=6$ のとき

$6^2 = 4^k(2\times 6+3)$

$36 = 4^k \times 15$

$4^k = \frac{36}{15} = \frac{12}{5}$

$\log_4 4^k = \log_4\frac{12}{5}$

$k = \log_4 \underline{\frac{12}{5}}$ クケ

[2].

(1) $0 \leq x < 2\pi$

$y = \cos 2x + 2\sin x + 1$

$\cos 2x = \underline{1 - 2\sin^2 x}$ アイ

$t = \sin x$ とおくと、 $-1 \leq t \leq 1$.

$y = 1 - 2t^2 + 2t + 1$

$= \underline{-2t^2 + 2t + 2}$ ウ〜オ.

$= -2(t^2 - t) + 2$

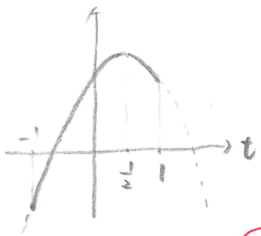
$= -2\left\{\left(t - \frac{1}{2}\right)^2 - \frac{1}{4}\right\} + 2$

$= -2\left(t - \frac{1}{2}\right)^2 + \frac{1}{2} + 2$

$= -2\left(t - \frac{1}{2}\right)^2 + \frac{5}{2}$

$t = \sin x = \frac{1}{2}$ $\quad x = \underline{\frac{\pi}{6}, \frac{5}{6}\pi}$ チ〜テ.

このとき、max $\underline{\frac{5}{2}}$ トナ.



$t = \sin x = -1.$ $\quad x = \underline{\frac{3}{2}\pi}$ ニヌ

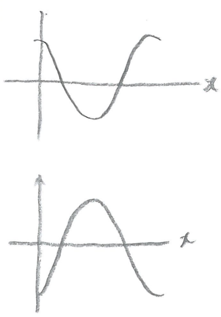
このとき、min $y = -2 - 2 + 2 = \underline{-2}$ ネノ

(2). $y = \sin x$ を y 軸方向へ $\underline{1}$ ハ

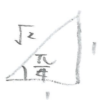
$\Rightarrow y = \sin x + 1$

$y = \cos x$ を x 軸で対称移動

$\Rightarrow y = -\cos x$ $\quad$ 4 ヒ



$\sin x + \cos x = \underline{\sqrt{2}}\sin\left(x + \underline{\frac{\pi}{4}}\right)$ フ ヘ



x 軸方向へ $-\frac{\pi}{4}$

$y = \sqrt{2}\sin\left(x + \frac{\pi}{4} + \frac{\pi}{4}\right) = \sqrt{2}\sin\left(x + \frac{\pi}{2}\right)$

$= \sqrt{2}\cos x$ $\quad \underline{1}$ ホ

[2]

$C: f(x) = -x^2 + 2x$

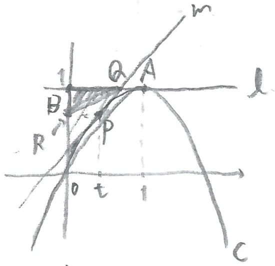
$= -(x^2 - 2x)$

$= -\{(x-1)^2 - 1\}$

$= -(x-1)^2 + 1$

$A(1, 1)$ アイ

$l: y = 1$ ウ



$S = \int_0^1 \{1 - (-x^2 + 2x)\} dx$

$= \int_0^1 (x^2 - 2x + 1) dx$

$= \left[\frac{1}{3}x^3 - x^2 + x\right]_0^1$

$= \frac{1}{3} - 1 + 1 = \frac{1}{3}$ エオ

$0 < t < 1$ $P(t, f(t))$

$f'(x) = -2x + 2$ カ〜ク

$m: y = f'(t)(x - t) + f(t)$

$= (-2t+2)(x-t) - t^2 + 2t$

$= (-2t+2)x + 2t^2 - 2t - t^2 + 2t$

$= (-2t+2)x + t^2$ ケ

$(-2t+2)x + t^2 = 1$

$2(t-1)x = t^2 - 1$

$2(t-1)x = (t-1)(t+1)$

$x = \frac{t+1}{2}$

$Q\left(\frac{t+1}{2}, 1\right)$ コサ

$B(0, 1)$, $R(0, t^2)$

$S_{PBR} = (1 - t^2) \times \frac{t+1}{2} \times \frac{1}{2}$

$= \frac{1}{4}(t+1)(1-t^2)$ シ〜ソ

$S_{Cm y} + S_{Clm} = S$

$S_{Cm y} = \int_0^t \{(-2t+2)x + t^2 - (-x^2 + 2x)\} dx$

$= \int_0^t (x^2 - 2tx + t^2) dx$

$= \left[\frac{1}{3}x^3 - tx^2 + t^2 x\right]_0^t$

$= \frac{1}{3}t^3 - t^3 + t^3 = \frac{1}{3}t^3$

$S_{Clm} = \int_t^{\frac{t+1}{2}} \{(-2t+2)x + t^2 - (-x^2 + 2x)\} + \int_{\frac{t+1}{2}}^1 \{1 - (-x^2 + 2x)\} dx$

$= \left[\frac{1}{3}x^3 - tx^2 + t^2 x\right]_t^{\frac{t+1}{2}} + \left[\frac{1}{3}x^3 - x^2 + x\right]_{\frac{t+1}{2}}^1$

$= \cancel{\frac{1}{3}\left(\frac{t+1}{2}\right)^3} - t\left(\frac{t+1}{2}\right)^2 + t^2 \cdot \frac{t+1}{2} - \frac{1}{3}t^3 + \frac{1}{3}$

$+ \left\{\cancel{\frac{1}{3}\left(\frac{t+1}{2}\right)^3} - \left(\frac{t+1}{2}\right)^2 + \frac{t+1}{2}\right\}$

$= (1-t)\left(\frac{t+1}{2}\right)^2 + (t^2 - 1)\frac{t+1}{2} - \frac{1}{3}t^3 + \frac{1}{3}$

$S = \cancel{\frac{1}{3}t^3} - \cancel{\frac{1}{3}t^3} - \frac{(t-1)(t+1)^2}{4} + \frac{(t^2-1)(t+1)}{2} + \frac{1}{3}$

$= \frac{(t^2-1)\{-(t+1) + 2(t+1)\}}{4} + \frac{1}{3}$

$= \frac{(t^2-1)(t+1)}{4} + \frac{1}{3} = \frac{t^3 + t^2 - t - 1}{4} + \frac{1}{3}$

$= \frac{1}{4}t^3 + \frac{1}{4}t^2 - \frac{1}{4}t + \frac{1}{12}$ タ〜ネ.

$S' = \frac{3}{4}t^2 + \frac{2}{4}t - \frac{1}{4} = 0$

$3t^2 + 2t - 1 = 0$

$(3t-1)(t+1) = 0$

$t = \frac{1}{3}, -1$ $0 < t < 1$ より $t = \frac{1}{3}$ ノハ.

min. $S\left(\frac{1}{3}\right) = \frac{1}{4} \times \frac{1}{27} + \frac{1}{4} \times \frac{1}{9} \cancel{- \frac{1}{4} \times \frac{1}{3} + \frac{1}{12}}$

$= \frac{1+3}{4 \cdot 27} = \frac{4}{4 \cdot 27} = \frac{1}{27}$ ヒ〜ヘ.

$\frac14 t_0^3+\frac14 t_0^2-\frac14 t_0+\frac1{12}=\frac14$

$3t_0^3+3t_0^2-3t_0+1=3$

$3t_0^3+3t_0^2-3t_0-2=0$

$t_0=0$のとき　-2

$t_0=\frac13$のとき

$3\cdot\frac{1}{27}+3\cdot\frac19-3\cdot\frac13-2$

$=\frac{1+3-9-18}{9}=-\frac{23}{9}$

$t=\frac23$のとき

$3\cdot\frac{8}{27}+3\cdot\frac49-3\cdot\frac23-2$

$=\frac{8+12-18-18}{9}=-\frac{16}{9}$

$t=1$のとき　$3+3-3-2=1$

$S=\frac14$をみたすt_0は

$\frac23<t_0<1$　4 ホ

(2). $0\le k\le n$ をみたすとき

D内の $y=k$ 上の点

$(0,k),(1,k),(2,k),\dots,(2n-2k,k)$ チ

より $(2n-2k+1)$コ

Dに含まれる格子点の数

$\sum_{k=0}^{n}(2n-2k+1)=2n+1+\sum_{k=1}^{n}(2n+1)-2\sum_{k=1}^{n}k$

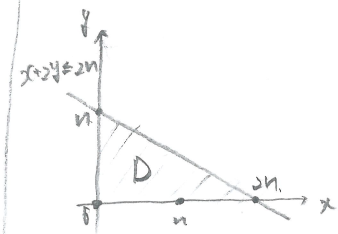
$=2n+1+(2n+1)n-2\cdot\frac{n(n+1)}{2}$

$=2n+1+2n^2+\cancel{n}-n^2-\cancel{n}$

$=n^2+2n+1$

$=(n+1)^2$

[3].



$x\ge0,\ y\ge0$

$x+2y\le2n$　$y\le-\frac12x+n$

$(0,0),(2n,0),(0,n)$　3 ア　2 イ

(1) $n=1$のとき

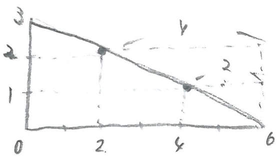
D:(0,0),(2,0),(0,1) 内

格子点 4コ ウ

$n=2$のとき

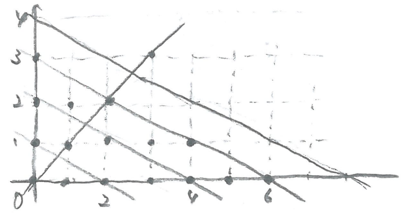
D:(0,0),(4,0),(0,2) 内

格子点 9コ エ



(3). n:自然数

$y\ge0,\ x+2y\le2n,\ y\le x$



$x+2y=2n$

$-)\ x-y=0$

$3y=2n$

$y=\frac23n=x$

E $(0,0),(2n,0),\left(\frac{2n}{3},\frac{2n}{3}\right)$

$y=k$の点の数

$2n-2k+1-k$

$=2n-3k+1$　セ〜タ

nが3の倍数　$n=3m$とすと、

$N=\frac23\times3m=2m$

$\sum_{k=0}^{N}(2n-3k+1)$

$=2n+1+(2n+1)N-3\cdot\frac{N(N+1)}{2}$

$=\frac{4n+2+(4n+2)N-3N^2-3N}{2}$

$=\frac{4n+2+4nN-3N^2-N}{2}$

$(N=\frac23n$より$)$

$=\frac{4n+2+4n\cdot\frac23n-3\cdot(\frac23n)^2-(\frac23n)}{2}$

$=\frac{12n+6+8n^2-4n^2-2n}{6}$

$=\frac{4n^2+10n+6}{6}=\frac{2n^2+5n+3}{3}$

$=\frac{(n+1)(2n+3)}{3}$　ツ〜ト

$n=3m-2$のとき

$\frac23(3m-2)=2m-\frac43$

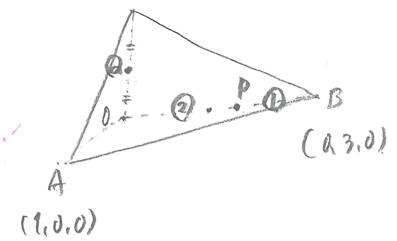
$N=2m-2$

$n=3m-1$のとき

$\frac23(3m-1)=2m-\frac23$

$N=2m-1$

[8].



$\begin{cases} \vec{OP} = \frac{2}{3}\vec{OB} \\ \vec{OQ} = \frac{1}{2}\vec{OC} \end{cases}$

$|\vec{OA}| = 1.\ |\vec{OB}| = 3.\ \angle AOB = 90°$

$|\vec{OC}| = \sqrt{1^2+1^2+\sqrt{2}^2} = \sqrt{1+1+2}$

$= \sqrt{4} = 2$ ア

$\vec{OA}\cdot\vec{OB} = 0$ イ

$\vec{OB}\cdot\vec{OC} = 0\times1+3\times1+0\times\sqrt{2} = 3$ ウ

$\vec{OC}\cdot\vec{OA} = 1\cdot1+1\times0+\sqrt{2}\times0 = 1$ エ

$P(0, 2, 0)$ オ. $Q\left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right)$ カ〜ク.

ABCの重心G.

$G\left(\frac{1+0+1}{3}, \frac{0+3+1}{3}, \frac{0+0+\sqrt{2}}{3}\right)$

$= G\left(\frac{2}{3}, \frac{4}{3}, \frac{\sqrt{2}}{3}\right)$ ケ〜ス

$\vec{OT} = k\vec{OG} = \frac{\vec{OA}+\vec{OB}+\vec{OC}}{3}k$ —①

$\vec{OT} = \vec{OA} + s\vec{AP} + t\vec{AQ}$

$= \vec{OA} + s(-\vec{OA}+\vec{OP}) + t(-\vec{OA}+\vec{OQ})$

$= \vec{OA} + s(-\vec{OA}+\frac{2}{3}\vec{OB}) + t(-\vec{OA}+\frac{1}{2}\vec{OC})$

$= (1-s-t)\vec{OA} + \frac{2}{3}s\vec{OB} + \frac{1}{2}t\vec{OC}$ —②

①②より

$\begin{cases} \frac{1}{3}k = 1-s-t & —③ \\ \frac{1}{3}k = \frac{2}{3}s & —④ \\ \frac{1}{3}k = \frac{1}{2}t & —⑤ \end{cases}$

④より $s = \frac{1}{2}k$.

⑤より $t = \frac{2}{3}k$.

③へ代入

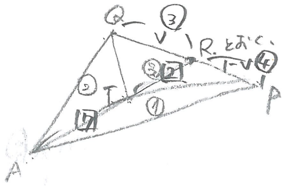
$\frac{1}{3}k = 1 - \frac{1}{2}k - \frac{2}{3}k$

$2k = 6 - 3k - 4k$

$9k = 6.\ k = \frac{2}{3}$ セソ

$s = \frac{1}{2}\times\frac{2}{3} = \frac{1}{3}$ タチ

$t = \frac{2}{3}\times\frac{2}{3} = \frac{4}{9}$ ツテ



$\vec{OT} = \vec{OA} + \frac{1}{3}\vec{AP} + \frac{4}{9}\vec{AQ}$

①:②:③

$\vec{AR} = u\vec{AT} = u(-\vec{OA}+\vec{OT}) = u(-\vec{OA}+\vec{OA}+\frac{1}{3}\vec{AP}+\frac{4}{9}\vec{AQ})$

$= \frac{1}{3}u\vec{AP} + \frac{4}{9}u\vec{AQ}$

$\vec{AR} = v\vec{AP} + (1-v)\vec{AQ}$

$\begin{cases} \frac{1}{3}u = v. \\ \frac{4}{9}u = 1-v \end{cases}$ $\quad \frac{4}{9}u = 1 - \frac{1}{3}u \quad \frac{7}{9}u = 1.$ $\quad u = \frac{9}{7}.\quad v = \frac{1}{3}\times\frac{9}{7} = \frac{3}{7}$

$\vec{AR} = \frac{3}{7}\vec{AP} + \frac{4}{9}\vec{AQ}$ より $QR : RP = 3 : 4$

$\vec{AR} = \frac{9}{7}\vec{AT}$ より $AT : TR = 7 : 2.$

① $\triangle APT = 1\times\frac{4}{7}\times\frac{7}{9} = \frac{4}{9}$

② $\triangle ATQ = 1\times\frac{3}{7}\times\frac{7}{9} = \frac{1}{3}$

③ $\triangle TPQ = 1-\left(\frac{4}{9}+\frac{1}{3}\right) = 1-\frac{7}{9} = \frac{2}{9}$

面積比 $\frac{4}{9} : \frac{1}{3} : \frac{2}{9} = 4 : 3 : 2$ ト〜ニ

OG上の点H, $AH \perp OG$

$\vec{OH} = l\vec{OG}$ とする. $\vec{AH}\cdot\vec{OG} = 0$ ヌ より

$(-\vec{OA}+\vec{OH})\cdot\vec{OG} = (-\vec{OA}+l\vec{OG})\cdot\vec{OG} = -\vec{OA}\cdot\vec{OG} + l|\vec{OG}|^2$

$= -\left(1\times\frac{2}{3}+0\times\frac{4}{3}+0\times\frac{\sqrt{2}}{3}\right) + l\left\{\left(\frac{2}{3}\right)^2+\left(\frac{4}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right\}$

$= -\frac{2}{3} + l\times\frac{4+16+2}{9} = -\frac{2}{3} + \frac{22}{9}l = 0$

$\frac{22}{9}l = \frac{2}{3},\quad l = \frac{3}{11}$ ネ〜ハ

OAPH.

$V = \left(\frac{2}{3}\times\frac{1}{3}\times\frac{3}{11}\right)W = \frac{2}{33}W$ ヒ〜ヘ

底面積比　高さ比.