第1回

I

[1]. (1) $\cos\frac{\pi}{3} = \frac{1}{2}$ ア,イ

$0 \leq x < 2\pi$ で $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3},\ \frac{5}{3}\pi$ ウ,エ

(2). $\sqrt{2}\sin 2x - 2a\sin x - \sqrt{6}\cos x + \sqrt{3}a = 0$ −(*)

$\sin 2x = 2\sin x\cos x$ オ

$2\sqrt{2}\sin x\cos x - 2a\sin x - \sqrt{6}\cos x + \sqrt{3}a = 0$

$2\sin x(\sqrt{2}\cos x - a) - \sqrt{3}(\sqrt{2}\cos x - a) = 0$

$(2\sin x - \sqrt{3})(\sqrt{2}\cos x - a) = 0$ カ キ ク

$\sin x = \frac{\sqrt{3}}{2}$, $\cos x = \frac{a}{\sqrt{2}}$

$\sin x = \frac{\sqrt{3}}{2}$ のとき $x = \frac{\pi}{3},\ \frac{2}{3}\pi$ ケ〜サ.

(*)が3個の異なる実数解をもつ.

(i) $2\sin x - \sqrt{3} = 0$ のとき.

$\sqrt{2}\cos x - a = 0$ となる.

$\cos x = \frac{a}{\sqrt{2}}$ をみたすxが

ただ1つの解となるのは.

$\cos x = \pm 1$ のとき すなわち.

$\frac{a}{\sqrt{2}} = \pm 1$. $a = \pm\sqrt{2}$.

(ii) $x = \frac{\pi}{3}$ のとき

$\cos\frac{\pi}{3} = \frac{a}{\sqrt{2}}$

$\frac{1}{2} = \frac{a}{\sqrt{2}}$ $a = \frac{\sqrt{2}}{2}$

$\cos x = \frac{1}{2}$. $x = \frac{\pi}{3},\ \frac{5}{3}\pi$ 3つ目.

(iii). $x = \frac{2}{3}\pi$ のとき

$\cos\frac{2}{3}\pi = \frac{a}{\sqrt{2}}$

$-\frac{1}{2} = \frac{a}{\sqrt{2}}$. $a = -\frac{\sqrt{2}}{2}$

$\cos x = -\frac{1}{2}$. $x = \frac{2}{3}\pi,\ \frac{4}{3}\pi$ 3つ目

よって $a = \pm\sqrt{2},\ \pm\frac{\sqrt{2}}{2}$ シ〜セ.

[2]. $f(x) = -2^x + (\sqrt{2})^{x+1} + 4$

$t = (\sqrt{2})^x$ とすると.

$f(x) = -\sqrt{2}^{2x} + \sqrt{2}(\sqrt{2})^x + 4$

$= -t^2 + \sqrt{2}t + 4$ ソ,タ.

(1). xが全ての実数 → $t > 0$. チ.

$f(t) = -t^2 + \sqrt{2}t + 4$ とすると

$f'(t) = -2t + \sqrt{2} = 0$

$2t = \sqrt{2}$

$t = \frac{\sqrt{2}}{2}$

t	0	…	$\frac{\sqrt{2}}{2}$	…
f'		−	0	+
f	4	↗	$\frac{9}{2}$	↘

$(\sqrt{2})^x = \frac{\sqrt{2}}{2} = \sqrt{2}\cdot\sqrt{2}^{-2} = (\sqrt{2})^{-1}$

$x = -1$.

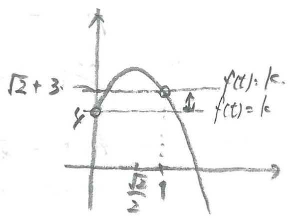
$f\left(\frac{\sqrt{2}}{2}\right) = -\left(\frac{\sqrt{2}}{2}\right)^2 + \sqrt{2}\cdot\frac{\sqrt{2}}{2} + 4$

$= -\frac{1}{2} + 1 + 4 = -\frac{1}{2} + 5 = \frac{9}{2}$

$x = -1$ で 最大値 $\frac{9}{2}$ ツテ, トナ.

(2). $x < 0$ のとき $0 < t < 1$. ニ.

$x > 0$ のとき $1 < t$



$4 < k < 3+\sqrt{2}$ ヌ〜ノ.

2.

$f(x) = x^3 - x.$

$f'(x) = 3x^2 - 1 = 0$　アイ

$3x^2 = 1,\quad x = \pm\frac{\sqrt{3}}{3}$

x	$\cdots$	$-\frac{\sqrt{3}}{3}$	$\cdots$	$\frac{\sqrt{3}}{3}$	$\cdots$
f'	+	0	−	0	+
f	↗	大	↘	小	↗

$x = \frac{\sqrt{3}}{3}$ ウエ　極大値　$f\left(\frac{\sqrt{3}}{3}\right) = \frac{1}{3\sqrt{3}} - \frac{1}{\sqrt{3}}$

$= \frac{1-3}{3\sqrt{3}} = \frac{-2\sqrt{3}}{9}$　オ〜ク

$x = \frac{-\sqrt{3}}{3}$ ケ〜サ　極小値　$f\left(-\frac{\sqrt{3}}{3}\right) = -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{9}$　シ〜セ

接点 $(1, f(1)) = (1, 0)$ において、

$f'(1) = 3 - 1 = 2$

接線 l:　$y = 2(x-1)$

$= 2x - 2$　ソタ

$g(x) = px^2 - qx \quad (p \neq 0)$

$g'(x) = 2px - q.$

$f(1) = g(1) = p - q = 0$　チ　$q = p.$

$f'(1) = g'(1) = 2p - q = 2$　ツテ

$2p - p = 2.$

$p = 2 \quad q = 2$　ト　ナ

$g(x) = 2x^2 - 2x.$

$f(x) = g(x)$ より

$x^3 - x = 2x^2 - 2x.$

$x^3 - 2x^2 + x = 0$

$x(x^2 - 2x + 1) = 0$

$x(x-1)^2 = 0$

$x = 0,\ 1$

もう1つ　$x = 0$　ニ

$A(t, f(t)),\ B(t, g(t)) \quad 0 < t < 1$

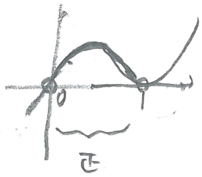
$f(t) - g(t)$

$= t^3 - t - (2t^2 - 2t)$

$= t^3 - 2t^2 + t = 0$

$t(t-1)^2 = 0$

$t = 0,\ 1.$



$0 < t < 1$ で

$f(t) - g(t) > 0$

$f(t) > g(t)$

② ヌ

$AB = \sqrt{(t-t)^2 + (f(t)-g(t))^2}$

$= f(t) - g(t)$

$= t^3 - 2t^2 + t$

$AB' = 3t^2 - 4t + 1 = 0$

$(3t-1)(t-1) = 0$

$t = \frac{1}{3},\ 1$

t	0	$\cdots$	$\frac{1}{3}$	$\cdots$	1
AB'		+		−	
AB		↗	大	↘	

$AB_{max} = \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \frac{1}{3}$

$= \frac{1}{27} - \frac{2}{9} + \frac{1}{3}$

$= \frac{1 - 6 + 9}{27} = \frac{4}{27}$　ネ〜ハ

$S = \int_0^1 (x^3 - 2x^2 + x)\,dx$

$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2\right]_0^1$

$= \frac{1}{4} - \frac{2}{3} + \frac{1}{2} = \frac{3 - 8 + 6}{12} = \frac{1}{12}$　ヒ〜ヘ

[3].

(1). $a_n = a_1 + (n-1)d$

$a_1 = 2 \quad a_2 = 5 \Rightarrow d = 3$ ア

$a_n = 2 + (n-1)\cdot 3$

$= 3n - 1$ イウ

$S_n = \dfrac{n(2+3n-1)}{2}$

$= \dfrac{n(3n+1)}{2}$

$= \dfrac{3}{2}n^2 + \dfrac{1}{2}n$. エ〜キ.

(2). $\sum_{k=1}^{n} b_n = T_n = n^2$

$b_1 = 1^2 = 1$ ク

$b_n = T_n - T_{n-1}$

② ケ ① コ

$b_n = n^2 - (n-1)^2$

$= n^2 - (n^2 - 2n + 1)$

$= 2n - 1$ サシ.

(3) 1群 b_1, b_2 | 2群 b_3, b_4, b_5, b_6, b_7 | 3群 $b_8, b_9 \cdots$

a_1コ a_2コ a_3コ

$c_1 = b_2$, $c_2 = b_{2+5} = b_7$ ス

$c_3 = b_{2+5+8} = b_{15}$ セソ

$c_k = b_{S_k} = 2\cdot S_k - 1$

$= 2\cdot\left(\dfrac{3}{2}k^2 + \dfrac{1}{2}k\right) - 1$

$= 3k^2 + k - 1$ タ〜ツ

$302 \times 12 = 604 + 302\,\,\to 3624$

$c_k \le 301$ となる最大の k は.

$3k^2 + k - 1 \le 301$

$3k^2 + k - 302 \le 0$

$k = \dfrac{-1 \pm \sqrt{1 + 4\cdot 3\cdot 302}}{6}$

$= \dfrac{-1 \pm \sqrt{3625}}{6}$

$k \geqq 1$ より. $1 \le k < \dfrac{-1+\sqrt{3625}}{6}$

$60 < \sqrt{3625} < 61$.

$59 < -1+\sqrt{3625} < 60$

$\dfrac{59}{6} < \dfrac{-1+\sqrt{3625}}{6} < 10$.

よって $c_k \le 301$ をみたす k の範囲は.

$1 \le k \le 9$ であるから

301 は、第10群 とわかる. テト.

・第9群の末項 c_9 は.

$81 \times 3 = 243$, $243 + 9 = 252$

$c_9 = b_{S_9}$

$S_9 = \dfrac{3}{2}\times 9^2 + \dfrac{1}{2}\times 9 = \dfrac{243+9}{2} = \dfrac{252}{2} = 126$.

$c_9 = b_{126}$.

・第10群の末項 c_{10} は.

$c_{10} = b_{S_{10}}$

$S_{10} = \dfrac{3}{2}\times 10^2 + \dfrac{1}{2}\times 10 = 150 + 5 = 155$

第10群の初項は b_{127}、末項は b_{155}

その総和は、

$\sum_{k=127}^{155} b_k = \sum_{k=1}^{155} b_k - \sum_{k=1}^{126}$ —①

$\sum_{k=1}^{155} b_k = \sum_{k=1}^{155}(2k-1) = 2\cdot\dfrac{155\cdot 156}{2} - 155$

$= 155(156-1) = 155^2$

$\sum_{k=1}^{126} b_k = \sum_{k=1}^{126}(2k-1) = 2\cdot\dfrac{126\cdot 127}{2} - 126$

$= 126(127-1) = 126^2$

$155 + 126 = 281$, $155 - 126 = 29$

①より.

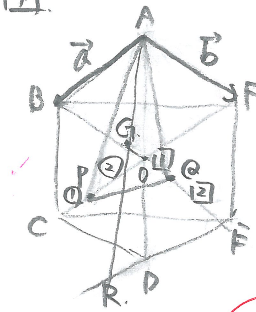
$\sum_{k=127}^{155} b_k = 155^2 - 126^2 = (155+126)\cdot(155-126)$

$= 281 \times 29 = 281\times(30-1)$

$= 8430 - 281 = 8149$ ナ〜ネ

$8430 - 281 = 8149$

[4]



$\vec{OP} = \frac{2}{3}\vec{AB} = \frac{2}{3}\vec{a}$ アイ

$\vec{OQ} = \frac{1}{3}\vec{AF} = \frac{1}{3}\vec{b}$ ウエ

$\vec{AO} = \vec{a}+\vec{b}$ より

$\vec{AP} = \vec{AO} + \frac{2}{3}\vec{OC}$

$= \vec{a}+\vec{b}+\frac{2}{3}\vec{a}$

$= \frac{5}{3}\vec{a}+\vec{b}$ オカ

$\vec{AQ} = \vec{AO}+\frac{1}{3}\vec{OE}$

$= \vec{a}+\vec{b}+\frac{1}{3}\vec{b}$

$= \vec{a}+\frac{4}{3}\vec{b}$ キク

$\vec{AG} = \frac{\vec{AP}+\vec{AQ}}{3}$

$= \frac{\frac{5}{3}\vec{a}+\vec{b}+\vec{a}+\frac{4}{3}\vec{b}}{3}$

$= \frac{5\vec{a}+3\vec{b}+3\vec{a}+4\vec{b}}{9}$

$= \frac{8}{9}\vec{a}+\frac{7}{9}\vec{b}$ ケ〜シ

$\vec{AR} = k\vec{AG} = \frac{8}{9}k\vec{a}+\frac{7}{9}k\vec{b}$ —①

$\vec{AR} = \vec{AD}+t\vec{a} = 2\vec{a}+2\vec{b}+t\vec{a}$

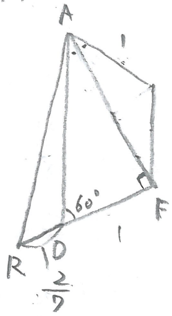
$= (\underset{ス}{2+t})\vec{a}+\underset{セ}{2}\vec{b}$ —②

①, ② より

$$\begin{cases} \frac{8}{9}k = 2+t \\ \frac{7}{9}k = 2 \end{cases}$$

$k = 2\times\frac{9}{7} = \frac{18}{7}$ ソ〜タ

$t = \frac{8}{9}\times\frac{18}{7}-2 = \frac{16}{7}-2 = \frac{2}{7}$ ツテ



$\vec{AR} = \frac{16}{7}\vec{a}+2\vec{b}$

$AF = \sqrt{3}$.

$S_{ADR} = \frac{1}{2}\times\frac{2}{7}\times\sqrt{3}$

$= \frac{\sqrt{3}}{7}$ トナ

$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\frac{2}{3}\pi = -\frac{1}{2}$ ニ〜ネ

$|\vec{AR}|^2 = \left(\frac{16}{7}\vec{a}+2\vec{b}\right)^2$

$= \frac{256}{49}|\vec{a}|^2+\frac{64}{7}\vec{a}\cdot\vec{b}+4|\vec{b}|^2$

$= \frac{256}{49}+\frac{64}{7}\times\left(-\frac{1}{2}\right)+4$

$= \frac{256-224+196}{49}$

$= \frac{228}{49}$

$|\vec{AR}| = \frac{2\sqrt{57}}{7}$ ノ〜フ

16×4 = 64
16×16 = 256
32×7 = 224
49×4 = 196
256 − 224 = 32
32 + 196 = 228
228 ÷ 2 = 114, 114 ÷ 2 = 57