

47回

[1]

[1] $a > 0$.

$$p: (x-2\sqrt{3})(x-2\sqrt{5}) > 0$$

$$\underline{x < 2\sqrt{3}, 2\sqrt{5} < x}$$

$$q: |x-4| > 1$$

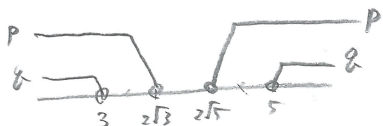
$$x-4 < -1, 1 < x-4$$

$$\underline{x < 3, 5 < x}$$

$$r: x < 2a \text{ かつ } x > \sqrt{5}a$$

(1) $x < \frac{3}{2}, x > \frac{5}{2}$

(2) $p \Rightarrow q$ かつ $q \Rightarrow p$ かつ $\frac{2}{3}$



$$2\sqrt{3} = \sqrt{12}, 2\sqrt{5} = \sqrt{20}$$

$$3 = \sqrt{9}, 5 = \sqrt{25}$$

(3) $A = \{x \mid x \text{ は } p \text{ かつ } q \}$

$B = \{x \mid x \text{ は } q \text{ かつ } r \}$

$C = \{x \mid x \text{ は } r \text{ かつ } p \}$

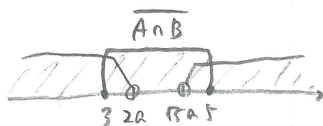
$$U = (\bar{A} \cup \bar{B}) \cup C$$

$$= \overline{A \cap B} \cup C$$

$$A \cap B = p \cap q = \emptyset$$

$$\overline{A \cap B} = \bar{\emptyset} = \{x \mid 3 \leq x \leq 5\}$$

$$(3 \leq x \leq 5) \text{ かつ } (x < 2a, \sqrt{5}a < x)$$



$$3 \leq 2a, \sqrt{5}a \leq 5$$

$$\frac{3}{2} \leq a, a \leq \frac{5}{\sqrt{5}} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\therefore \frac{3}{2} \leq a \leq \sqrt{5}$$

[2]

$$G_1: y = -x^2 + 4ax - 4a^2 - 8a + 52$$

$$= -(x^2 - 4ax) - 4a^2 - 8a + 52$$

$$= -(x - 2a)^2 + 4a^2 - 4a^2 - 8a + 52$$

$$= -(x - 2a)^2 - 8a + 52$$

$$G_2: y = x^2 - 4(a+1)x + 8a + 4$$

$$= \{x - 2(a+1)\}^2 - 4(a+1)^2 + 8a + 4$$

$$= \{x - 2(a+1)\}^2 - 4a^2 - 8a - 4 + 8a + 4$$

$$= \{x - 2(a+1)\}^2 - 4a^2$$

(1) G_1 の頂点 $P(2a, -8a + 52)$

$$G_1 \text{ が } x \text{ 軸と 2 点で交わる}$$

$$-8a + 52 > 0$$

$$8a < 52$$

$$a < \frac{52}{8} = \frac{13}{2}$$

$$x^2 - 4(a+1)x + 8a + 4$$

$$= (x-2)(x-4a-2)$$

$$\begin{array}{r} 1 \quad 4(2a+1) \quad -4a-4 \\ 1 \quad -2(2a+1) \quad -(4a+2) \\ 1 \quad -2 \quad -2 \end{array}$$

$$G_2 \text{ が } x \text{ 軸と 2 点で交わる}$$

$$4a + 2 \neq 2$$

$$4a \neq 0$$

$$a \neq \frac{0}{4}$$

(2) $a > 0$, G_2 が x 軸と 2 点で交わる Q, R とする

$$a > 0 \text{ かつ } 4a > 0, 4a + 2 > 2$$

$$Qx = 2, Rx = 4a + 2 \quad (Qx < Rx)$$

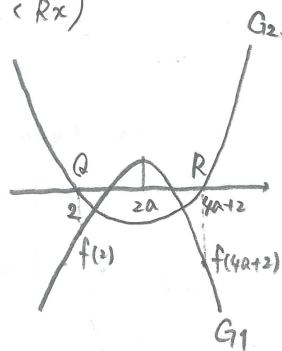
$$G_1: y = f(x) \text{ とする}$$

$$2 < 2a < 4a + 2 \quad \text{--- (1)}$$

$$f(2) \leq 0 \quad \text{--- (2)}$$

$$f(4a+2) \leq 0 \quad \text{--- (3)}$$

$$a < \frac{13}{2} \text{ (注) } \quad \text{--- (4)}$$



① 5/1

$$1 < a$$

$$2a < 4a + 2$$

$$-2 < 2a \quad -1 < a$$

$$\therefore 1 < a \quad \text{--- (1')}$$

② 5/2

$$f(2) = -4 + 8a - 4a^2 - 8a + 52$$

$$= -4a^2 + 48 \leq 0$$

$$a^2 - 12 \geq 0$$

$$(a + 2\sqrt{3})(a - 2\sqrt{3}) \geq 0$$

$$a \leq -2\sqrt{3}, \quad 2\sqrt{3} \leq a \quad \text{--- (2')}$$

③ 5/1

$$f(4a+2) = -(4a+2-2a)^2 - 8a+52$$

$$= -(2a+2)^2 - 8a+52$$

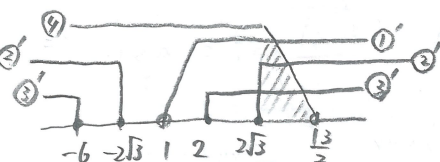
$$= -4a^2 - 8a - 4 - 8a + 52$$

$$= -4a^2 - 16a + 48 \leq 0$$

$$a^2 + 4a - 12 \geq 0$$

$$(a+6)(a-2) \geq 0$$

$$a \leq -6, \quad 2 < a \quad \text{--- (3')}$$

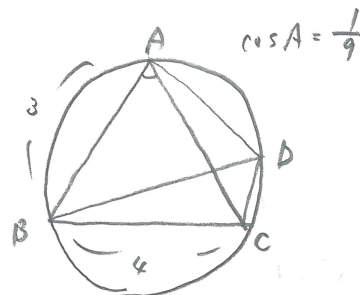


$$2\sqrt{3} \leq a < \frac{13}{2} \quad \text{--- (4')}$$

②

(1)

$$\sin A = \sqrt{1 - \frac{1}{81}} = \sqrt{\frac{80}{81}} = \frac{\sqrt{80}}{9}$$



$$\frac{\sqrt{80}}{9} = \frac{\sqrt{20}}{3}$$

$$CA = x \quad \text{--- (1)}$$

$$\frac{x^2 + 3^2 - 4^2}{2 \times x \times 3} = \frac{1}{9}$$

$$\frac{3}{1} = \frac{-2}{-3} = \frac{-2}{-9}$$

$$x^2 + 9 - 16 = \frac{2}{3}x$$

$$x^2 - \frac{2}{3}x - 7 = 0$$

$$3x^2 - 2x - 21 = 0$$

$$(x-3)(3x+7) = 0$$

$$x = 3, \quad -\frac{7}{3}$$

$$x > 0 \quad \therefore CA = 3 \quad \text{--- (2')}$$

$$\sin \angle ABD : \sin \angle CBD = 3 : 1 \quad \text{--- (3')}$$

$$\sin \angle ABD = k$$

$$\sin \angle CBD = k$$

$$\text{--- (4')}$$

$$\text{正弦定理 --- (5')}$$

$$\frac{AD}{\sin \angle ABD} = \frac{CD}{\sin \angle CBD}$$

$$\frac{AD}{3k} = \frac{CD}{k}$$

$$AD = 3CD \quad \text{--- (6')}$$

$$\text{余弦定理 --- (7')}$$

$$\cos \angle ABC = \frac{3^2 + 4^2 - 5^2}{2 \times 3 \times 4} = \frac{2}{3}$$

$$\cos \angle ADC = \frac{AD^2 + CD^2 - 3^2}{2 \times AD \times CD}$$

$$\cos(180^\circ - \angle ABC) = \frac{(3CD)^2 + CD^2 - 9}{2 \times 3CD \times CD}$$

$$-\cos \angle ABC = \frac{10CD^2 - 9}{6CD^2}$$

$$-\frac{2}{3} = \frac{10CD^2 - 9}{6CD^2}$$

$$-4CD^2 = 10CD^2 - 9$$

$$14CD^2 = 9$$

$$CD^2 = \frac{9}{14}$$

$$CD = \sqrt{\frac{9}{14}} = \frac{3}{\sqrt{14}}$$

$$= \frac{3\sqrt{14}}{14} \quad \text{--- (8')}$$

[2]. 30人.

- $\begin{cases} \#1 \text{ 回分位: } 8 \text{ 人用.} \\ \#2 \text{ " : } 15, 16 \text{ 人用} \\ \#3 \text{ " : } 23 \text{ 人用} \end{cases}$

(1) 30人平均.

$$\frac{2 \times 0.3 + 8 \times 0.6 + 12 \times 1.0 + 5 \times 1.4 + 3 \times 2.0}{30}$$

$$= \frac{0.6 + 4.8 + 12 + 7 + 6}{30}$$

$$= \frac{30.4}{30} = 1.01... \approx \frac{1.0}{1} \text{ 26}$$

(2). 23人用 D: $\frac{3}{1}$

(3). 全員 0.1 入す.

①: 全員の平均値 0.1 入す。

①: 各階級の平均値を化回
化かする。x

②: 分散は変化しない。

②: 1回分位階級 ⑤
は変化する。

④: 3回分位階級 D → C
へ変化がわかる。x

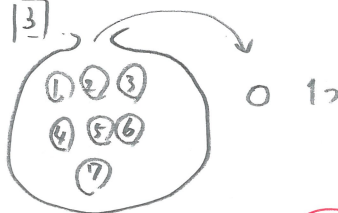
$\frac{1}{1} \times 9 \text{ 分}$

[3].

	0.5以下	全件	割合
A	0	2	0
B	2	8	$\frac{2}{8} = \frac{1}{4} = 0.25$
C	4	12	$\frac{4}{12} = \frac{1}{3} = 0.33$
D	2	5	$\frac{2}{5} = 0.4$
E	1	3	$\frac{1}{3} = 0.33$

$\frac{3}{1}$

[3].



(1) 1回目⑤から $\frac{1}{7}$

2回目⑤から $\frac{6}{7} \times \frac{1}{6} = \frac{1}{7}$

1回目④, 2回目⑥ $\frac{1}{7} \times \frac{1}{6} = \frac{1}{42}$

(2). [(A) ⑤から出た.]
[(B) ④と⑥から出た.] $\Rightarrow$ 2回目終了

1回目終了 $\Rightarrow$ (A) $\frac{1}{7}$

2回目終了
[(A) 2回目終了] $\frac{6}{7} \times \frac{1}{6} = \frac{1}{7}$

[(B) 2回目終了] $\frac{1}{42} \times 2 = \frac{1}{21}$

5.2. $\frac{1}{7} + \frac{1}{21} = \frac{3+1}{21} = \frac{4}{21}$

3回目以降終了

$$1 - \left(\frac{1}{7} + \frac{4}{21} \right) = 1 - \frac{3+4}{21} = 1 - \frac{7}{21}$$

$$= \frac{14}{21} = \frac{2}{3}$$

4回目終了

(i) (A) 2回目終了

(a) 1~3回目④⑤⑥から出る

$$\frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{35}$$

(b) 1~3回目④⑤⑥から出る, ④⑤は出る

$$\frac{1}{7} \times \frac{4}{6} \times \frac{3}{5} \times 3 \times \frac{1}{4} \times 2 = \frac{3}{35}$$

(ii) (B) 2回目終了

1~3回目④⑤⑥から出る, ④⑤は出る

$$(b) \text{と同じ} \Rightarrow \frac{3}{35}$$

$$5.2. \text{ 4回目終了の確率: } \frac{1}{35} + \frac{3}{35} + \frac{3}{35} = \frac{7}{35} = \frac{1}{5}$$

4回目終了の確率, ④⑤⑥から出る確率

$$P(X) = \frac{1}{5}, \quad P(X \cap Y) = \frac{3}{35} + \frac{3}{35} = \frac{6}{35}$$

$$P_X(Y) = \frac{P(X \cap Y)}{P(X)} = \frac{\frac{6}{35}}{\frac{1}{5}} = \frac{6}{7}$$

[4]

(1) $P = 124(5)$

$$P = 1 \times 5^2 + 2 \times 5 + 4 \times 1$$

$$= 25 + 10 + 4 = \underline{39} \times 1$$

$$3P = 3(1 \times 5^2 + 2 \times 5 + 4 \times 1)$$

$$= 3 \times 5^2 + 6 \times 5 + 12$$

$$= 3 \times 5^2 + 6 \times 5 + 2 \times 5 + 2$$

$$= 3 \times 5^2 + \underline{8 \times 5} + 2$$

$$= 3 \times 5^2 + (5+3) \times 5 + 2$$

$$= 3 \times 5^2 + 1 \times 5^2 + \underline{\frac{9 \times 5}{2}} + 2$$

$$= 4 \times 5^2 + 3 \times 5 + 2.$$

$$3P + 25 = 4 \times 5^2 + 3 \times 5 + 2 + 25$$

$$= 4 \times 5^2 + 3 \times 5 + 2 + 1 \times 5^2$$

$$= 5 \times 5^2 + 3 \times 5 + 2$$

$$= 1 \times 5^3 + 3 \times 5 + 2.$$

$$= \underline{1032}(5)$$

$7 \sim 2.$

(2). $1 \leq a \leq 4, 0 \leq b \leq 4, 1 \leq c \leq 4.$

$$N = abc(5)$$

$$= a \times 5^2 + b \times 5 + c$$

$$= \underline{25a + 5b + c}$$

$\frac{42}{7}$

(i) $N = cba(7) \times 5.$

$$= c \times 7^2 + b \times 7 + a$$

$$= 49c + 7b + a$$

$$N = cba(7) = abc(5)$$

$$49c + 7b + a = 25a + 5b + c.$$

$$2b = 24a - 48b$$

$$b = 12a - 24b$$

$$= \underline{12(a - 2b)}$$

$\sim$

$$0 \leq b \leq 4 \times 1$$

$$b = 12(a - 2c) = 0.$$

$$a - 2c = 0.$$

$$a = 2c$$

$$1 \leq a \leq 4, 1 \leq c \leq 4 \times 1$$

$$(a, b, c) = \underline{(2, 0, 1)}, \underline{(4, 0, 2)}$$

$\sim$

(ii) $1 \leq d \leq 4.$

$$3N = dca(5)$$

$$= 25d + 5c + a$$

$$N = 25a + 5b + c \times 1$$

$$3(25a + 5b + c) = 25d + 5c + a.$$

$$75a + 15b + 3c = 25d + 5c + a.$$

$$74a - 2c = 25d - 15b.$$

$$2(37a - c) = 5(5d - 3b)$$

$$37a - c = 5k.$$

$$c = 37a - 5k.$$

$$a = 1 \text{ or } 2, \quad c = 37 - 5k.$$

$$k = 7 \text{ or } 8, \quad c = 37 - 35 = 2.$$

$$a = 2 \text{ or } 3, \quad c = 74 - 5k.$$

$$k = 14 \text{ or } 15, \quad c = 74 - 70 = 4. \quad \text{Not possible.}$$

5.2. $(a, c) = \underline{(1, 2)} \times k$

$$5d - 3b = 2k, \quad k = 7 \times 1$$

$$5d - 3b = 2 \times 7 = 14$$

$$d = 4, b = 2 \text{ or } 3, \quad 5 \times 4 - 3 \times 2 = 20 - 6 = 14$$

$$(a, b, c, d) = (1, 2, 2, 4)$$

$$3N = dca(5) \times 7 \quad 3N = 421(5)$$

$$1000(5) \leq 3N + 5m \leq 4444(5)$$

$$1000(5) \leq 421(5) + 5m \leq 4444(5)$$

$$24(5) \leq 5m \leq 4023(5)$$

$$2 \times 5 + 4 \leq 5m \leq 4 \times 5^3 + 2 \times 5 + 3$$

$$10 + 4 \leq 5m \leq 500 + 10 + 3$$

$$14 \leq 5m \leq 513.$$

$$\frac{14}{5} \leq m \leq \frac{513}{5}$$

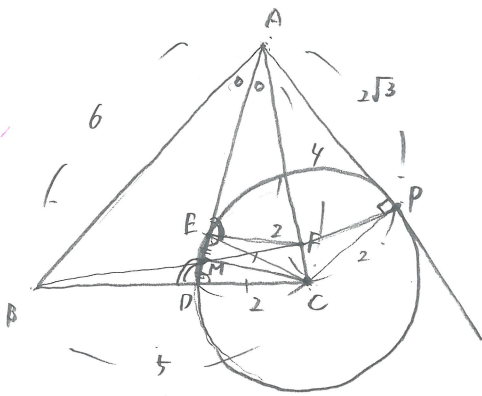
$$3 \leq m \leq 102$$

$$102 - 3 + 1 = 100$$

$$\underline{100} \times 1 \times 2$$

$$\begin{array}{r} \times 565 \\ 421 \\ \hline 54 \\ 4454 \\ 821 \\ \hline 4023 \end{array}$$

5)



$$(1) \frac{BD}{DC} = \frac{6}{4} = \frac{3}{2} \text{ r.}$$

$$DC = \frac{2}{5} BC = \frac{2}{5} \times 5 = 2$$

$$CP = 2, AC = 4, \angle APC = 90^\circ \text{ r.}$$

$$AP = \sqrt{4^2 - 2^2} = \sqrt{16 - 4} = \sqrt{12} = 2\sqrt{3}$$

$$(2) \triangle ABD \sim \triangle ACE \text{ r.}$$

$$AE : AD = AC : AB$$

$$AE : AD = 4 : 6 = 2 : 3$$

$$3AE = 2AD$$

$$AE = \frac{2}{3} AD$$

また r.

$$AE \cdot AD = AP^2$$

$$\frac{2}{3} AD \cdot AD = (2\sqrt{3})^2$$

$$\frac{2}{3} AD^2 = 12$$

$$AD^2 = 12 \times \frac{3}{2} = 18$$

$$AD = 3\sqrt{2}$$

$$DM : ME = 1 : 1 \text{ r.}$$

$$DM : ED = 1 : 2$$

$$DM : ME : ED = 1 : 1 : 2$$

$$AE : AD = 2 : 3 \text{ r.}$$

$$AE : ED : AD = 2 : 1 : 3 \\ = 4 : 2 : 6$$

$$AE : EM : MD = 4 : 1 : 1$$

$$\text{また r. } AM : MD = (AE + EM) : MD \\ = (4 + 1) : 1 \\ = 5 : 1$$

$$BD : DC : BC = 3 : 2 : 5$$

また r.

$$\frac{FA}{CF} \cdot \frac{MD}{AM} \cdot \frac{BC}{DB} = 1 \quad \left\{ \begin{array}{l} \frac{FA}{CF} = 3 \end{array} \right.$$

$$\frac{FA}{CF} \cdot \frac{1}{5} \cdot \frac{5}{3} = 1 \quad \left\{ \begin{array}{l} CF : FA = 1 : 3 \end{array} \right.$$

$\triangle CDE$ は $CD = CE = 2$ の等腰三角形で $\angle C = 90^\circ$ r.

DE の中点 M と C を結ぶ線分 CM は

$CM \perp DE$ となる。

$$AE : AD = 2 : 3 \text{ r. } DE = \frac{1}{3} AD = \frac{1}{3} \times 3\sqrt{2} = \sqrt{2}$$

$$DM = \frac{1}{2} DE = \frac{1}{2} \times \sqrt{2} = \frac{\sqrt{2}}{2}$$

$$CM = \sqrt{2^2 - \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{4 - \frac{2}{4}} = \sqrt{\frac{14}{4}} = \frac{\sqrt{14}}{2}$$

$$\bullet \triangle ACE \text{ の面積 } S_{ACE} = \frac{1}{2} \times AE \times CM$$

$$\text{① } AE = \frac{2}{3} \times AD = \frac{2}{3} \times 3\sqrt{2} = 2\sqrt{2} \text{ r.}$$

$$S_{ACE} = \frac{1}{2} \times 2\sqrt{2} \times \frac{\sqrt{14}}{2} = \frac{2\sqrt{7}}{2} = \sqrt{7}$$

$$\bullet \triangle AEF \text{ の面積 } S_{AEF}$$

$$AF : FC = 3 : 1 \text{ r.}$$

$$S_{AEF} = \frac{3}{4} S_{ACE} = \frac{3}{4} \times \sqrt{7} = \frac{3\sqrt{7}}{4} \quad \text{--- ①}$$

$$\bullet \triangle ACP \text{ の面積 } S_{ACP} = \frac{1}{2} \times AP \times CP$$

$$S_{ACP} = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3}$$

$$\bullet \triangle AFP \text{ の面積 } S_{AFP}$$

$$S_{AFP} = \frac{3}{4} S_{ACP} = \frac{3}{4} \times 2\sqrt{3} = \frac{3\sqrt{3}}{2} \quad \text{--- ②}$$

$$\text{①② r. } \text{四角形 } AEPF \text{ の面積} = S_{AEF} + S_{AFP}$$

$$= \frac{3\sqrt{7}}{4} + \frac{3\sqrt{3}}{2} = \frac{6\sqrt{3} + 3\sqrt{7}}{4} \text{ r.}$$